FLOW IN A UNIFORMLY EXPANDING LAYER AND EXPANSION OF A GAS VOLUME INTO VACUUM

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In this paper we shall propose an approximate model of uniformly expanding layers which, under certain conditions, reduces three-dimensional unsteady flow problem to certain one- or two-dimensional problems.

Consider the expansion into a vacuum of a gas jet in which the kinetic energy of the particles is large in comparison with the gas internal energy. Let the initial flow in the jet be one-dimensional (with plane or cylindrical waves) and assume that the distribution of velocity, density and pressure along the axis of the jet is given. When the energy ratio is as assumed above, the subsequent decay of pressure to zero will have little effect on the velocity of the particles in the direction of the initial motion. Only the transverse expansion of the jet will be significant. Thus it is natural to make the following simplifying assumption: from a certain instant of time t_0 on, the gas particles move under their own momentum and each particle conserves its velocity.

Under this assumption, consecutive layers of particles in the jet do not interact with each other and their motions are independent. Thus it is sufficient to solve the problem of the expansion of a gas in an infinitely thin layer cut out from the jet. In this case the pressure and the density distribution across the layer will be equalized instantaneously and the velocity distribution across the layer will be linear. Such a layer expands uniformly.

1. The system of coordinates for a two-dimensional uniformly expanding layer is given in Fig. 1. In view of the assumption made above, we have

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} = \frac{\partial \rho}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = 0 \quad (1.1)$$

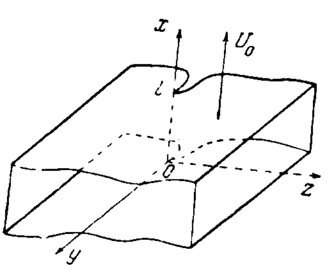

Fig. 1

Without loss of generality, one of the bounding planes of the layer can be the stationary plane $x = 0$. Let the other plane move in the x direction with the velocity u_0 and let its position at $t = 0$ be $x = l$. Under the present assumptions, the particles of the layer move in the x direction under their own momentum. The velocity gradient in the x direction is then

$$\frac{\partial u}{\partial x} = \frac{u_0}{l + u_0 t} = \frac{1}{t + \omega}, \qquad \omega = \frac{l}{u_0} \quad (1.2)$$

The velocity of the particles in the layer

$$u = \frac{x}{t + \omega} \tag{1.3}$$

Thus the u component of velocity is specified. This leads to a significant simplification of the three-dimensional unsteady adiabatic flow equations. The equations of motion and the adiabatic condition will be the same as for two-dimensional flow (in the yz plane)

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \qquad \frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{1.4}$$

and the equation of continuity differs from that for two-dimensional flow only by the presence of a time-dependent term

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}\right) + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{t+\omega} = 0 \tag{1.5}$$

The system of coordinates for a cylindrical uniformly expanding layer is shown in Fig.2. The surface of the inner cylinder has an initial radius x_1 and expands with the velocity u_1. The corresponding values for the outer cylinder are x_2 and u_2. We shall consider only one-dimensional flows in the layer, for which the v velocity is parallel to the generatrix of the cylinder. Furthermore, let us restrict the law of expansion of the layer by imposing the additional condition

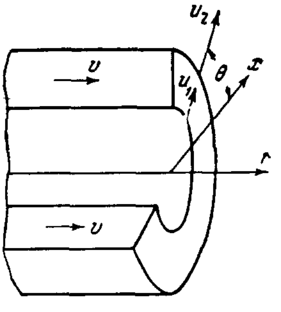

Fig. 2

$$\frac{x_1}{u_1} = \frac{x_2}{u_2} = \frac{x_2 - x_1}{u_2 - u_1} = \omega \tag{1.6}$$

In cylindrical coordinates (x, θ, r) this gives

$$\frac{\partial v}{\partial x} = \frac{\partial \rho}{\partial x} = \frac{\partial p}{\partial x} = \frac{\partial u}{\partial r} = 0, \qquad \frac{\partial}{\partial \theta} = 0 \tag{1.7}$$

The u velocity of inertial flow in cylindrical expansion, which satisfies the boundary conditions $u = u_1$ at $x = x_1 + u_1 t$, $u = u_2$ at $x = x_2 + u_2 t$ and condition (1.6) is

$$u = \frac{x}{t+\omega} \tag{1.8}$$

In the case of one-dimensional flow in the direction of the r-axis in a uniformly expanding layer, the equation of motion and the adiabatic condition are the same as in the case of flow with plane waves, and the equation of continuity is

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r}\right) + \frac{\partial v}{\partial r} + \frac{2}{t+\omega} = 0 \tag{1.9}$$

The equations of one-dimensional adiabatic gas flow in a uniformly expanding layer can be written in the general form

$$\begin{gathered}\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \qquad \frac{\partial}{\partial t}\frac{p}{\rho^\gamma} + v\frac{\partial}{\partial r}\frac{p}{\rho^\gamma} = 0 \\ \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu - 1)\frac{\rho v}{r} + \lambda\rho\frac{1}{t+\omega} = 0\end{gathered} \tag{1.10}$$

Here $\nu = 1, 2$ stands for flows with plane and cylindrical waves, and $\lambda = 1, 2$ for flows in plane and cylindrical layers, respectively.

The cases in which these equations can be solved are the following:

a) Monatomic gas, $\gamma = 5/3$. This case can be solved by means of the transformation [1 and 2]

$$\tau = a - \frac{b^2}{t-d}, \quad \eta = \frac{b}{t-d} y, \quad \zeta = \frac{b}{t-d} z, \quad v' = \frac{t-d}{b} v - \frac{y}{b}$$
$$w' = \frac{t-d}{b} w - \frac{z}{b}, \quad \rho' = \left(\frac{b}{t-d}\right)^3 \rho, \quad p' = \left(\frac{b}{t-d}\right)^5 p \tag{1.11}$$

which is an invariant transformation for three-dimensional unsteady gas flows. One can easily verify that for $d = -\omega$ transformation (1.11) transforms Equations (1.4) and (1.5) and Equations (1.10) with $\nu = 1$, $\lambda = 2$ into the equations of two- and one-dimensional gas flow, respectively.

Thus, transformation (1.11) can be used to transform any solution for two-dimensional unsteady flow with $\gamma = {}^5/_3$ into a solution for a uniformly expanding plane layer and any one-dimensional plane-wave solution into a one-dimensional flow in a cylindrical uniformly expanding layer.

b) Flow with velocity proportional to a coordinate [3]. It can be easily verified that

$$v = -\frac{1}{\mu}\frac{d\mu}{dt} r, \qquad p = [A + B\varphi(\mu r)] \frac{\mu^{\nu\gamma}}{(t+\omega)^{\lambda\gamma}}$$
$$\rho = \frac{\mu^{\nu-1}}{r}(t+\omega)^{-\lambda}\varphi'(\mu r), \qquad \mu'' - \frac{2\mu'^2}{\mu} = B\frac{\mu^{3+\nu(\gamma-1)}}{(t+\omega)^{\lambda(\gamma-1)}} \tag{1.12}$$

which is analogous to L.I.Sedov's solution [3], is an exact solution of (1.10).

Here $\varphi(\mu r)$ is an arbitrary function and A, B are arbitrary constants. Solution (1.12) is a special case of L.V.Ovsiannikov's solution [4], in which the velocity components are proportional to the corresponding coordinates.

c) Isothermal flow of a gas, $\gamma = 1$. In this case Equations (1.4), (1.5) or (1.10) can be trnsformed by means of the transformation

$$p = p'(t+\omega)^{-\lambda}, \qquad \rho = \rho'(t+\omega)^{-\lambda} \tag{1.13}$$

into the ordinary equations of two- or one-dimensional isothermal gas flow.

Note that Equations (1.10) contain a parameter ω with the dimension of time, so that self-similar solutions cannot be generalized for the case of a uniformly expanding layer.

2. In the following we shall consider the solutions for certain expansion flows in a uniformly expanding layer.

Consider an expansion flow in which the velocity is proportional to the coordinate [3]. In the case of cylindrical symmetry with $p/\rho^\gamma = \text{const}$ and $\gamma = {}^5/_3$ this solution is [6]

$$v' = \frac{1}{R(\tau)}\frac{dR}{d\tau} r' = \pm\frac{3}{\sqrt{2}}\sqrt{R^{4/3} - 1}\,\frac{1}{R^{5/3}} r', \qquad \rho = \frac{1}{R^2(\tau)}\left(1 - \frac{r^2}{R^2}\right)^{3/2} \tag{2.1}$$

Here we have introduced the dimensionless variables and Functions (*)

$$\tau = \frac{c_1'\tau_0}{r_1'}, \qquad r' = \frac{r_0'}{r_1'}, \qquad v' = \frac{v_0'}{c_1'}, \qquad \rho' = \frac{\rho_0'}{\rho_1'} \tag{2.2}$$

where c_1', ρ_1' are the velocity of sound and the density at $\tau = 0$, $r' = 0$; r_1' is the radius of the volume occupied by the gas at the time $\tau = 0$. The plus and minus signs in (2.1) correspond to expanding and converging flow, respectively. The function $R(\tau)$ for $\gamma = {}^5/_3$ is given in [5](Fig. 102, $\nu = 2$). Some values of R are

$\tau = 0$	0.04	0.17	0.45	0.6	1.0	2.0
$R = 1.0$	1.002	1.04	1.28	1.47	2.08	3.92

Let us introduce the dimensionless variables for a uniformly expanding flow

*) From here on the subscript o shall denote the dimensional variables which enter (1.11).

$$t = \frac{c_1 t_0}{r_1}, \quad r = \frac{r_0}{r_1}, \quad v = \frac{v_0}{c_1}, \quad \rho = \frac{\rho_0}{\rho_1} \tag{2.3}$$

Here c_1, ρ_1 are the velocity of sound and the density at $t = 0$, $r = 0$; r_1 is the radius of the volume occupied by the gas at $t = 0$.

The transformation formulas (1.11) for the dimensionless variables (2.2) and (2.3) are

$$\tau = a - \frac{b^2}{t+k}, \quad r' = \frac{b}{m}\frac{r}{t+k}, \quad v = \frac{mb}{t+k}v' + \frac{r}{t+k}, \quad \rho = \left(\frac{mb}{t+k}\right)^3 \rho' \tag{2.4}$$

where $k = \omega c_1/r_1$ is a known constant, and a, b, m are arbitrary constants (*)

Solution (2.1), transformed for the case of a uniformly expanding layer, is

$$v(r, t) = \left[\frac{1}{t+k} \pm \left(\frac{b}{t+k}\right)^2 \frac{3}{\sqrt{2}} \frac{\sqrt{R^{4/3}-1}}{R^{5/3}}\right] r$$

$$\rho(r, t) = \left(\frac{mb}{t+k}\right)^3 \frac{1}{R^2}\left[1 - \frac{1}{m^2}\left(\frac{b}{t+k}\right)^2 \frac{r^2}{R^2}\right]^{3/2}, \quad R = R(\tau) = R\left(a - \frac{b^2}{t+k}\right) \tag{2.5}$$

Consider, for the case of a uniformly expanding layer, the solution which corresponds to the initial conditions

$$v(r, 0) = 0, \qquad \rho(1, 0) = 0, \qquad \rho(0, 0) = 1 \tag{2.6}$$

Substitution of (2.5) into the first condition in (2.6) yields

$$\pm \frac{3}{\sqrt{2}} \frac{b^2}{k} \sqrt{R_0^{4/3} - 1} + R_0^{5/3} = 0, \qquad R_0 = (R)_{t=0} = R\left(a - \frac{b^2}{k}\right) \tag{2.7}$$

This condition can be satisfied only if the first term is taken with the minus sign. Consequently, this sign should be taken in Equations (2.1) and (2.5). This means that the initial flow (2.1) represents a flow which converges towards the axis of symmetry.

The second and third condition in (2.6) lead to the relations

$$b^2 / m^2 k^2 R_0^2 = 1, \qquad (mb / k)^3 = R_0^2 \tag{2.8}$$

Equations (2.7),(2.8) determine the relation between the constants m, b, R_0 and the parameter k, viz.

$$m = \left(1 + \frac{2}{9k^2}\right)^{-1/8}, \qquad b = k\left(1 + \frac{2}{9k^2}\right)^{5/8}, \qquad R_0 = \left(1 + \frac{2}{9k^2}\right)^{3/4} \tag{2.9}$$

Using the relation $R(\tau)$ one can find [5] the value τ_0 which corresponds to R_0, and the value (**)

$$a = \tau_0 + \frac{b^2}{k} = \tau_0 + k\left(1 + \frac{2}{9k^2}\right)^{5/4} \tag{2.10}$$

The following are some values of $a(k)$ calculated for $0 \leqslant k \leqslant 1.3$:

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*) If we designate the constants in transformation (1.11) by the subscript o, then

$$k = -\frac{c_1 d_0}{r_1}, \quad a = \frac{c_1'}{r_1'} a_0, \quad b = \left(\frac{c_1 c_1'}{r_1 r_1'}\right)^{1/2} b_0, \quad m = \left(\frac{r_1' c_1'}{r_1 c_1}\right)^{1/2}$$

In real cases one can specify only the parameters c_1, r_1 and $\omega = -d_0$ of the flow in a uniformly expanding layer. The parameters of the initial two-dimensional flow c_1', r_1', which undergoes the trnasformation, are arbitrary, and so are the parameters a_0, b_0 of transformation (1.11).

**) As the initial condition represents a converging flow, the variable τ_0 should be taken with the minus sign.

$k = 0$	0.05	0.1	0.2	0.3	0.5	1.0	1.3
$a = -0.634$	−0.315	−0.17	0.013	0.154	0.4	0.95	1.27

Thus, the expansion flow of a gas in a uniformly expanding two-dimensional layer, which satisfies initial conditions (2.6) is determined from Equations (2.5) with the minus sign and $k = \omega c_1/r_1$, and with the constants a, b, m given by Equations (2.9),(2.10).

The one-dimensional plane-wave flow for $p/\rho^{\gamma} = \text{const}$, $\gamma = {}^5/_3$, with velocity proportional to the coordinate is [3 and 6]

$$v' = \frac{1}{R_1(\tau)}\frac{dR_1}{d\tau}r' = -3\frac{\sqrt{R_1^{2/3}-1}}{R_1^{4/3}}r', \qquad \rho' = \frac{1}{R_1(\tau)}\left(1-\frac{r'^2}{R_1^2}\right)^{3/2} \tag{2.11}$$

The function $R_1(\tau)$ for $\gamma = {}^5/_3$ is given in [5] (Fig.102, $\nu = 1$) and can be written in the form [6]

$$\tau = \pm\frac{R_1^{2/3}+2}{3}\sqrt{R_1^{2/3}-1} \tag{2.12}$$

Equations (2.2) to (2.4) hold for this case. The parameter

$$k = x_1c_1/r_1v_1 = \omega c_1/r_1 \tag{2.13}$$

Solution (2.11), transformed for a uniformly expanding layer, is

$$v(r,t) = \left[\frac{1}{t+k} - \left(\frac{b}{t+k}\right)^2\frac{3}{R_1^{4/3}}\sqrt{R_1^{2/3}-1}\right]r \tag{2.14}$$

$$\rho(r,t) = \left(\frac{mb}{t+k}\right)^3\frac{1}{R_1}\left[1-\frac{1}{m^2}\left(\frac{b}{t+k}\right)^2\frac{r^2}{R_1^2}\right]^{3/2}, \quad R_1 = R_1(\tau) = R_1\left(a-\frac{b^2}{t+k}\right)$$

Consider the uniformly expanding layer solution which satisfies the initial conditions

$$v(r,0) = 0, \qquad \rho(1,0) = 0, \qquad \rho(0,0) = 1 \tag{2.15}$$

According to (2.14) these conditions are

$$3b^2\sqrt{R_{10}^{2/3}-1} = kR_{10}^{4/3}, \qquad b^2 = m^2k^2R_{10}^2, \qquad m^3b^3 = k^3R_{10}$$

respectively.

These equations determine the relation between the constants m, b, R_{10} and the parameter k

$$m = \left(1+\frac{1}{9k^2}\right)^{-1/2}, \qquad b = k\left(1+\frac{1}{9k^2}\right), \qquad R_{10} = \left(1+\frac{1}{9k^2}\right)^{3/2} \tag{2.16}$$

In this case τ_0 is given by (2.12) as an explicit function of R_{10} and, consequently, it is an explicit function of k, viz.

$$\tau_0 = a - \frac{b^2}{k} = -\frac{1}{3k} - \frac{1}{81k^3}$$

$$a(k) = k - \frac{1}{3k} \tag{2.17}$$

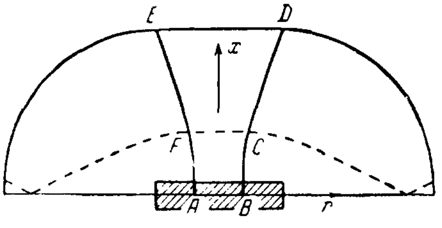

Fig. 3

Thus, the expansion flow of a gas in a uniformly expanding cylindrical layer with $\gamma = {}^5/_3$ and initial conditions (2.15) is determined by Equations (2.14) and relations (2.13), (2.16) and (2.17).

3. A characteristic feature of gas expansion into a vacuum is the rapid conversion of potential energy into kinetic energy of the particles.

In the case of many asymmetric volumes the initial stages of the expansion are sufficiently simple and can be calculated. Thus, if a portion of the surface of the volume is a plane (parallelepiped, disk, finite cone, etc.), then the initial stage of the expansion of that part of the volume which adjoins that surface is a one-dimensional plane-wave flow, which is subsequently eroded due to the effect of the boundaries of the plane surface. If a part of the surface is a circular cylinder or a sphere, then the initial stage of the expansion of the adjoining part of the volume is the corresponding one-dimensional flow with cylindrical or spherical symmetry.

Fig.3 represents the initial stage of an expansion flow. This pattern can be regarded as the upper half of the flow in a meridional cross-section of a disk or in a diametral cross-section of a circular cylinder. The shaded area represents the initial volume, occupied by the gas.

The part $ABDE$ repesents a one-dimensional jet, with plane waves in the case of a disk, and with cylindrical symmetry in the case of a circular cylinder. The flow inside $ABDE$ is known from the existing solutions of one-dimensional problems of expansion into a vacuum. One can easily determine, for any instant of time, the shape of the boundaries AE, BD of the region influenced by the edges of the disk or by the end faces of the cylinder. Thus, for every instant of time, one can calculate the mass and the kinetic and internal energies of the gas contained in the one-dimensional jet.

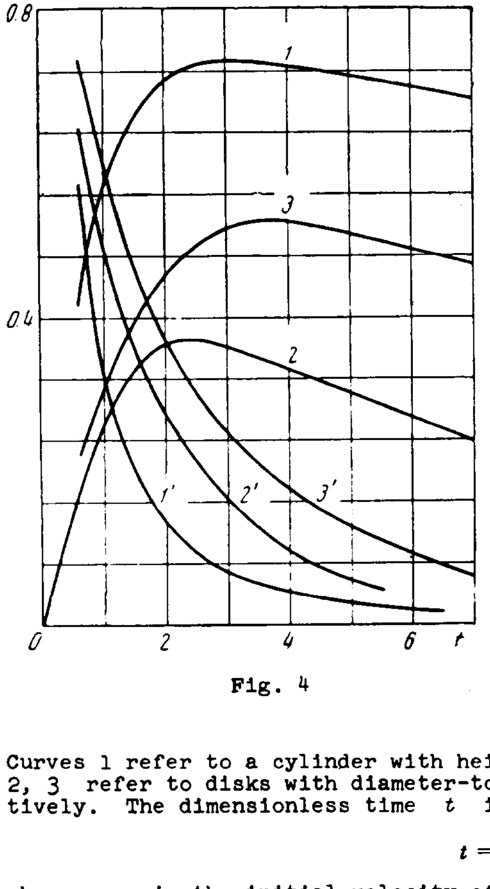

Fig. 4

The results of calculations for a thin disk and for a cylinder with large height-to-diameter ratio l with $\gamma = 5/3$ are given in Fig.4. The kinetic energy (curves 1, 2, 3) and the internal energy (curves 1′, 2′, 3′) are taken relative to the total initial energy of the whole gas volume. Curves 1 refer to a cylinder with height-to-diameter ratio $l = 5$ and curves 2, 3 refer to disks with diameter-to-thickness ratios of 7 and 15, respectively. The dimensionless time t is defined as

$$t = \frac{c_* t_0}{h} \tag{3.1}$$

where c_* is the initial velocity of sound at the axis or plane of symmetry of the initial volume and $2h$ is the thickness of the disk or the diameter of the cylinder. In the case of the disk the initial distribution of the gas parameters was assumed to be uniform, and in the case of the cylinder the initial distribution was such that the velocities were proportional to the coordinate (so-called self-similar distribution [3 to 6].

The last assumption was due to the absence of an analytic solution for cylindrical expansion in the case of uniform initial data and is justified by the fact [7] that in the one-dimensional expansion into a vacuum of initially stationary volumes of gas the asymptotic flow is weakly dependent on the initial distribution and is determined by the total energy E, mass M, and entropy function p/ρ^γ of the gas. As mentioned above, and as can be seen from Fig.4, the asymptotic character of the one-dimensional flow is attained very rapidly. Therefore expansion flows with uniform initial parameter distributions can be approximated with high accuracy by flows with self-similar initial distributions, where the relations between the characteristic parameters (characteristic density, velocity of sound, and dimension) are determined for the two flows possessing the same energy, mass and entropy.

For flows with plane waves and $\gamma = {}^5/_3$ these relations are

$$r_1 = 1.29r_2, \quad c_1 = 1.095c_2, \quad \rho_1 = 1.315\rho_2 \tag{3.2}$$

For flows with cylindrical symmetry and $\gamma = {}^5/_3$ these are

$$r_1 = 1.228r_2, \quad c_1 = 1.181c_2, \quad \rho_1 = 1.652\rho_2 \tag{3.3}$$

Here the subscripts 2 and 1 refer to the parameters of the uniform distribution and to the initial values of the parameters of the self-similar distribution at the plane or axis of symmetry, respectively.

Calculation of the volumes occupied by the one-dimensional and three-dimensional parts of the flow, in conjunction with the data of Fig.4, indicates that the mean mass and energy densities in the jets are significantly higher than in the remaining part of the volume occupied by the gas. This suggests the idea that the mass and energy in the three-dimensional part of the flow may be neglected and that the flow may be approximated by the expansion into the vacuum of a one-dimensional jet which exists at some instant t_0.

It follows from the data of Fig.4 that, from a certain instant of time on, the kinetic energy in the jet considerably exceeds the internal energy of the gas, i.e. the pressure energy. But in that case the calculation of the subsequent expansion into a vacuum of such jets can be carried out by the method for uniformly expanding layers which has been described above.

Let the calculation of the expansion of a one-dimensional jet begin at the time $T = t_0$, and let the coordinates of the gas particles at that time be $x = \xi$. All parameters of the one-dimensional flow at the time t_0 can be expressed in terms of ξ and t_0 by known formulas. At subsequent times $t = T - t_0$, according to the assumption stated above, the particle velocity in the x direction does not vary and, consequently,

$$x = \xi + u(\xi, t_0)\, t \tag{3.4}$$

This relation yields Equation

$$\xi = \xi(x, t; t_0) \tag{3.5}$$

As we are considering the expansion of an aggregate of layers which constitute a one-dimensional flow at the time t_0, then all constants entering solution (2.5) become functions of ξ and, in view of (3.5), functions of x and t. It is also necessary to pass from the dimensionless variables of a layer (2.3) to the dimensionless variables of a disk

$$t = \frac{c_* t_0}{h}, \quad x = \frac{x_0}{h}, \quad r = \frac{r_0}{h}, \quad v = \frac{v_0}{c_*}, \quad \rho = \frac{\rho_0}{\rho_*}, \quad c = \frac{c_0}{c_*} \tag{3.6}$$

Finally, in view of the fact that solution (2.5) corresonds not to a uniform flow but to a self-similar initial distribution, one must take into account the transformation multipliers which follow from relations (3.2) and (3.3). The expansion into a vacuum of a disk is determined by the system of functions

$$\rho(x, r, t) = \left(\frac{mbr_1}{t+\omega}\right)^3 \frac{1}{R^2}\left[1 - \frac{1}{m^2c_1^2}\left(\frac{b}{t+\omega}\right)^2 \frac{r^2}{R^2}\right]^{3/2}$$

$$v(x, r, t) = \left[\frac{1}{t+\omega} - \frac{3}{\sqrt{2}}\left(\frac{b}{t+\omega}\right)^2 \frac{r_1}{c_1}\frac{\sqrt{R^{4/3}-1}}{R^{5/3}}\right] r \tag{3.7}$$

$$u(x, t) = \frac{(x-\xi)}{t}, \qquad R = R\left(a - \frac{r_1}{c_1}\frac{b^2}{t+\omega}\right)$$

Here a, b and m depend, according to (2.9), on k, and k, r_1, c_1, and ω are, in turn, functions of $\xi = \xi(x, t; t_0)$ according to (3.5).

An analogous solution for the expansion into a vacuum of a circular cylinder, obtained on the basis of (2.14) by the method of uniformly expanding layers, is

$$\rho(x, r, t) = \left(\frac{mbr_1}{t+\omega}\right)^3 \frac{1}{R_1}\left[1 - \frac{1}{m^2 c_1^2}\left(\frac{b}{t+\omega}\right)^2 \frac{r^2}{R_1^2}\right]^{3/2}$$

$$v(x, r, t) = \left[\frac{1}{t+\omega} - 3\left(\frac{b}{t+\omega}\right)^2 \frac{r_1}{c_1} \frac{\sqrt{R_1^{2/3} - 1}}{R_1^{4/3}}\right] r \qquad (3.8)$$

$$u(x, t) = \frac{x}{t+\omega}, \qquad R_1(\tau) = R_1\left(a - \frac{r_1}{c_1}\frac{b^2}{t+\omega}\right)$$

The dependence of r_1, c_1, ω and u on $\xi(x, t; t_0)$ is determined from known solutions for the corresponding one-dimensional flows and can be significantly simplified by appropriate approximations.

The choice of the instant t_0 at which one begins the calculation of the transverse expansion of the jet is limited by a relatively narrow range of values (Fig.4) for which, on one hand, the one-dimensional part of the flow contains a large part of the total mass and energy as possible and, on the other hand, the potential energy is sufficiently small as compared with the kinetic energy. This range lies to the right of the value t_* at which the kinetic energy attains its maximum.

Taking into account the fact that the actual expansion of the one-dimensional jet takes place in a region of three-dimensional flow with a finite density (which has been neglected in the present method), one may expect that solutions obtained for $t > t_*$ are a lower bound for the actual asymptotic distribution of density in the gas.

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